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AGRICULTURAL MEMORANDUM NO 858

A THEORETICAL APPROACH TO THE USE OF MEAN MAXIMUM AND MINIMUM TEMPERATURES IN THE CALCULATION OF ONTARIO UNITS

The Ontario Unit consists of a day term (Y) and a night term (X).

where $Y = \alpha(MAX - 10) - \beta(MAX - 10)^2$ for MAX > 10°C, Y = 0 for MAX < 10°C - 1

and $X = \gamma(MIN - 4.4)$ for MIN > 4.4°C, X = 0 for MIN < 4.4°C - 2

MAX = day maximum air temperature
MIN = night minimum air temperature

$\alpha = 3.33$ $\beta = 0.084$ $\gamma = 1.8$

The Ontario Unit (OU) = (Y + X)/2. - 3

Daily contributions of the above are usually summed to give monthly values. Attempts have been made to use monthly mean maximum and minimum temperatures to give a mean daily value from which a monthly value is calculated. This method is most inaccurate when the maximum and minimum temperatures are around their base values (monthly mean temperature around 7°C). Substituting a mean maximum value of 10°C and mean minimum temperature of 4.4°C in the formulae for X and Y results in a value of zero for Ontario Units. Since maximum and minimum temperatures would have been above their base on about half the days in the month some units would have accumulated. In addition to the monthly mean values of maximum and minimum temperatures account must therefore be taken of the variability of daily values about the monthly means.

Considering the Y term above.

Let M be the monthly mean of the daily maximum temperatures
Let x = MAX - M be the departure of a daily value from the monthly mean
Let f(x) be the probability density of the distribution of x

We may then write the mean value of Y for a month as

$$\bar{Y} = \int_{x = 10 - M}^{\infty} \left\{ \alpha(x + M - 10) - \beta(x + M - 10)^2 \right\} f(x)\, dx \qquad - 4$$

Since values of x < 10 - M make no contribution to the sum.

If we assume that daily values of maximum temperature have a normal distribution about the monthly mean maximum with standard deviation σ then:-

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2) \qquad - 5$$

Rearranging 4 and substituting for f(x) gives -

$$\bar{Y} = \left\{\alpha(M-10) - \beta(M-10)^2\right\}\frac{1}{\sigma\sqrt{2\pi}}\int_{x=10-M}^{\infty} e^{-x^2/2\sigma^2}\,dx + \left\{\alpha - 2\beta(M-10)\right\}\frac{1}{\sigma\sqrt{2\pi}}\int_{x=10-M}^{\infty} x\,e^{-x^2/2\sigma^2}\,dx - \frac{\beta}{\sigma\sqrt{2\pi}}\int_{x=10-M}^{\infty} x^2 e^{-x^2/2\sigma^2}\,dx$$

(A) (B) (C)

Term A may be evaluated directly from tables of the probability integral (PI).
Term B may be directly integrated to give

$$\frac{\sigma}{\sqrt{2\pi}}\left\{\alpha - 2\beta(M-10)\right\}\exp\left(-(10-M)^2/2\sigma^2\right)$$

Term C may be integrated by parts to give

$$-\beta(10-M)\frac{\sigma}{\sqrt{2\pi}}\exp\left(-(10-M)^2/2\sigma^2\right) - \frac{\beta\sigma^2}{\sigma\sqrt{2\pi}}\int_{x=10-M}^{\infty} e^{-x^2/2\sigma^2}\,dx$$

the second part of which may be evaluated from tables of the PI.

Rearranging the terms A, B and C gives

$$Y = \left\{\alpha(M-10) - \beta(M-10)^2 - \beta\sigma^2\right\}\left[1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t=\frac{10-M}{\sigma}} e^{-t^2/2}\,dt\right]$$

$$+\frac{\sigma}{\sqrt{2\pi}}\left\{\alpha - \beta(M-10)\right\}e^{-\frac{(10-M)^2}{2\sigma^2}}$$

Similarly for X if the daily minimum temperature has a standard deviation σ_1 about the monthly mean (M1) then substituting γ for α and letting $\beta = 0$

$$X = \left\{\gamma(M_1 - 4.4)\right\}\left[1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t=\frac{4.4-M_1}{\sigma_1^2}} e^{-t^2/2}\,dt\right] + \frac{\sigma_1}{\sqrt{2\pi}}\left\{\gamma\, e^{-\frac{(4.4-M_1)^2}{2\sigma_1^2}}\right\}$$

The Integral is the normal distribution function and can be found from tables as $P_N\left(\gamma < \frac{\text{BASE-MEAN}}{\text{SD}}\right)$

Figure 1 shows the daily correction to be applied to the daytime term for various standard deviations of maximum temperature. As would be expected maximum corrections are at the base temperature of 10°. The correction becomes asymtopic to $-\beta\sigma^2$ as $\frac{M-10}{\sigma}$ increases.

Figure 2 shows the daily correction to be applied to the night-time term for various SDs of minimum temperature. Again maximum corrections are at the base temperature and are symmetrical about this point.

Both corrections show a discontinuity at the base temperature.

To illustrate best this method of calculating OUs let us assume typical values of the SDs of 2.0°C for both maximum and minimum temperatures about their respective monthly

means of 10.0°C and 4.4°C.

Standard methods would give X = Y = 0 ∴ OU = 0

From figures 1 and 2 X = 1.44 Y = 2.49

Monthly total of OUs = $\frac{(1.44 + 2.49)}{2}$ 31 = 60.9

From figure 1 it is expected that standard methods would give an overestimate of total OUs at higher mean maximum temperatures (say > 12.5 for σ = 2).

However standard deviations are not usually available (and their calculation would entail as much effort as calculating OUs from daily values). Highest maximum and minimum and lowest maximum and minimum are more readily available and fair estimates can be made by multiplying the range by a factor (a_n) to give $\hat{\sigma}$ Brooks and Carruthers "Range of values in a series of observation", Met Mag, London, 76, 1947, p 258). Temperatures are not independent variables and so the number of observations has to be scaled by a persistence factor.

Recent work suggests a value of 1.7, giving 18 independent variables for a month.

From standard statistical tables a_{18} = 0.260

and for weekly values a_4 = 0.486

Using the example of Long Ashton, October 1974 (AGMEM 848) we have

σ max = 1.52 σ min = 2.46

$\hat{\sigma}$ max = 1.88 $\hat{\sigma}$ min = 2.44

MEAN MAX = 11.3 MEAN MIN = 5.1 MONTHLY MEAN 8.2

METHOD	DAY TERM	NIGHT TERM	OU
Uncorrected monthly values	4.2	1.26	84.6
Corrected-AGMEM 848	6.043	1.813	121.8
Using correct σ	4.56	2.58	110.7
Using $\hat{\sigma}$ from range	4.84	2.45	113.0
Calculated from daily values			112.3

The accuracy of the method outlined above based on range is being tested and will be the subject of another AGMEM.

Any comments or suggestions on this paper should be referred to the author.

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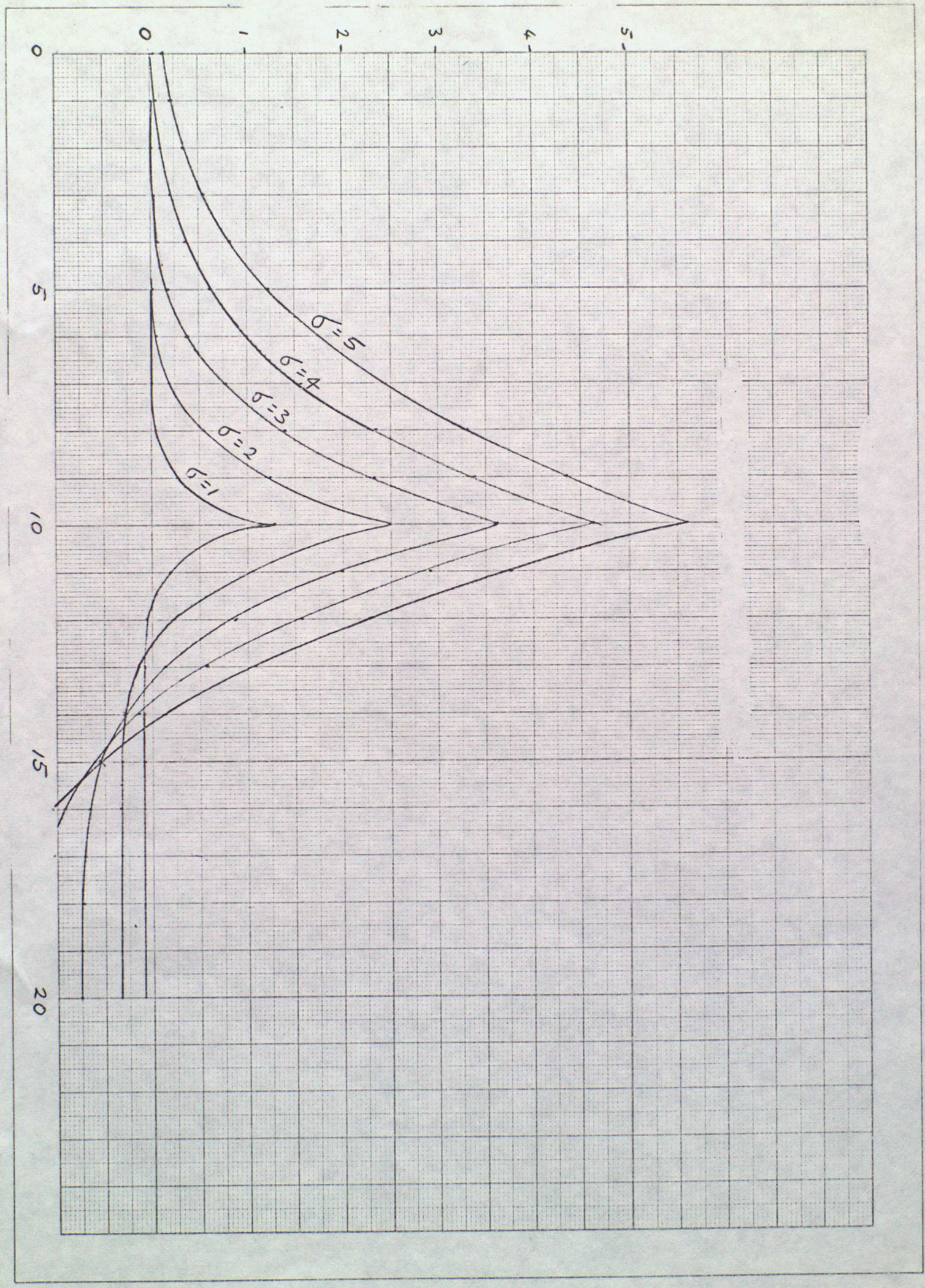
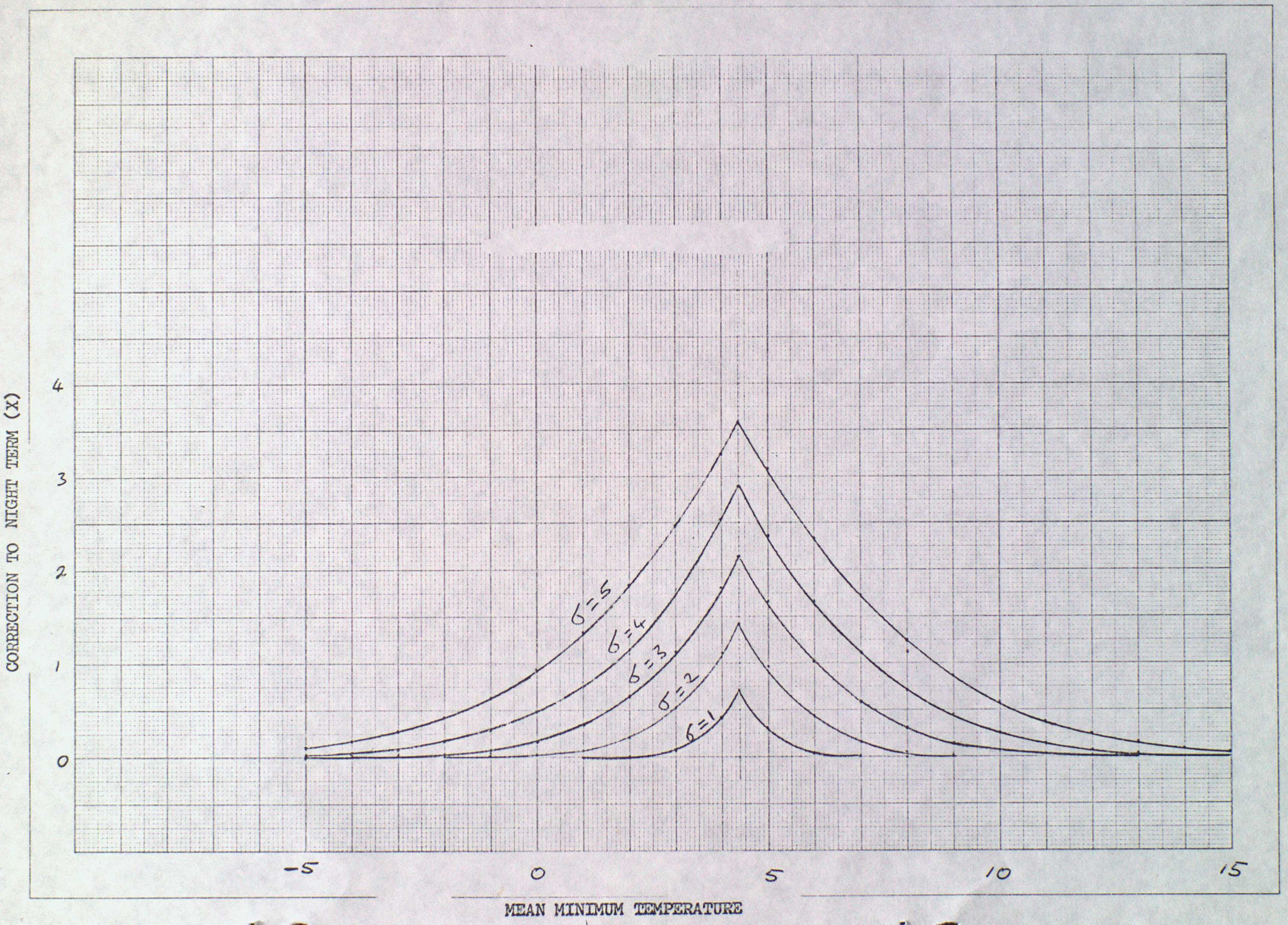

CORRECTION TO DAY TERM (Y)
0
1
2
3
4
5
0
5
10
15
20
MEAN MAXIMUM TEMPERATURE
σ=5
σ=4
σ=3
σ=2
σ=1

FIG 1

FIG 2


CORRECTION TO NIGHT TERM (X)
4
3
2
1
0
σ=5
σ=4
σ=3
σ=2
σ=1
-5
0
5
10
15
MEAN MINIMUM TEMPERATURE